Trend-Cycle Decomposition of Economic Activity in the Czech Republic

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Abstract. The aim of the paper is to decompose two important economic variables (GDP and unemployment rate in the Czech Republic) into a cyclical and a trend component by applying a state space methodology. An unobserved component model is econometrically estimated by the method of maximum likelihood. The likelihood function is constructed using the square root version of the Kalman filter. The results are economically interpreted and it is found that (1) the cyclical component of output and unemployment rate has already recovered from an initial shock at the beginning of the economic crisis in 2008, (2) there has been a persistently decreased growth of the trend component of output after the outbreak of the economic crisis, (3) the trend component of unemployment rate has been constant during the current crisis which suggests that possible hysteresis effects have not played an important role yet, (4) the growth of the GDP trend component is highly volatile in the Czech Republic.

Keywords: State space approach · Unobserved components model · Trend-cycle decomposition · Economic crisis

1 Introduction

There was a considerable decline in economic activity at the beginning of the current economic crisis in 2008 in the Czech Republic. The goal of the presented paper is to shed some light on the following questions: (1) has economic activity already recovered from the initial decline and to what extent, (2) is there evidence of permanently increased unemployment rate or not, (3) is long-run growth of GDP permanently decreased? Bivariate unobserved component model of GDP and unemployment rate is formulated in this paper in order to answer these questions. The model will be written in state space form which is a methodology commonly applied not only in technical sciences but also in economics (Zeng, Wu [17]). The model will be estimated by the method of maximum likelihood. The likelihood function is constructed using the square root version of the Kalman filter which has better numerical properties compared to the basic form of the filter (Anderson, Moore [1], Chui, Chen [5]).

Trend-cycle decomposition methodology has a long tradition in macroeconometrics (Nelson [14]) and is described in detail in Dagum and Bianconcini [7]. Empirical papers discussing trend-cycle decomposition of economic activity include Cerra and Saxena [4] who discuss this issue using regime switching methodology. Ball [2] estimates trend component of GDP for OECD countries by applying the concept of

R. Silhavy et al. (eds.), *Applied Computational Intelligence and Mathematical Methods*,
Advances in Intelligent Systems and Computing 662, DOI 10.1007/978-3-319-67621-0_2

potential output and a production function approach. Ball [2] and Barro [3] (and many others cited in these papers) found evidence that deep recessions have permanent effects on output. Similar results are found in the presented paper for the case of the Czech Republic.

The paper is organized as follows. Econometric methodology is described in Sect. 2. The subsequent Sect. 3 is the empirical part of the paper presenting results and economic discussion. The final Sect. 4 concludes.

2 Econometric Methodology

2.1 Model

The model decomposing real GDP and unemployment rate into trend and a cycle component is presented in this chapter. The applied unobserved components model was formulated by Clark [6] and was also summarized in a textbook treatment by Kim and Nelson [12]. The model equations are given as follows:

$$y_t = n_t + x_t \tag{1}$$

$$n_t = g_{t-1} + n_{t-1} + v_t,\ \ v_t \sim i.i.d.N\left(0, \sigma_v^2\right), \tag{2}$$

$$g_t = g_{t-1} + w_t,\ w_t \sim i.i.d.N\left(0, \sigma_w^2\right), \tag{3}$$

$$x_t = \phi_1 \cdot x_{t-1} + \phi_2 \cdot x_{t-2} + e_t,\ e_t \sim i.i.d.N\left(0, \sigma_e^2\right), \tag{4}$$

where y_t is log of real GDP, n_t is a stochastic trend component, x_t represents a stationary cyclical component and v_t, w_t, e_t are independent white noise processes.

The autoregressive process of order two was chosen in the Eq. (4). This is the most common assumption used in empirical literature when modelling cyclical variables as an autoregressive process of the second order is a parsimonious way to model cyclical dynamics.

This standard univariate model is extended into a bivariate model of real GDP and unemployment. The unemployment rate is decomposed into trend and a cycle as follows:

$$U_t = L_t + C_t, \tag{5}$$

$$L_t = L_{t-1} + \varepsilon_t,\ \varepsilon_t \sim i.i.d.N\left(0, \sigma_\varepsilon^2\right), \tag{6}$$

$$C_t = \alpha_0 \cdot x_t + \alpha_1 \cdot x_{t-1} + \alpha_2 \cdot x_{t-2} + \eta_t,\ \eta_t \sim i.i.d.N\left(0, \sigma_\eta^2\right), \tag{7}$$

where L_t is a trend component of unemployment rate, C_t is a stationary component of unemployment rate and ε_t, η_t are independent white noise processes.

The cyclical component C_t is assumed to be a function of current and past transitory components of real output which represents a version of Okun's law. The number of lags used in the Eq. (7) was chosen rather arbitrarily. This choice, however, is quite common in the empirical literature (Kim, Nelson [12]).

The transition and measurement equation of the state space representation are written as follows

$$\mathbf{x_t} = \mathbf{A} \cdot \mathbf{x_{t-1}} + \mathbf{u_t}, \tag{8}$$

$$\mathbf{z_t} = \mathbf{D} \cdot \mathbf{x_t} + \mathbf{v_t}, \tag{9}$$

where $\mathbf{x_t} = [n_t \quad x_t \quad x_{t-1} \quad x_{t-2} \quad g_t \quad L_t]'$, $\mathbf{z_t} = [y_t \quad U_t]'$,

$$\mathbf{A} = \begin{bmatrix} \mathbf{e_1} + \mathbf{e_5} \\ \phi_1 \cdot \mathbf{e_2} + \phi_2 \cdot \mathbf{e_3} \\ \mathbf{e_2} \\ \mathbf{e_3} \\ \mathbf{e_5} \\ \mathbf{e_6} \end{bmatrix}, \mathbf{D} = \begin{bmatrix} \mathbf{e_1} + \mathbf{e_2} \\ \alpha_0 \cdot \mathbf{e_2} + \alpha_1 \cdot \mathbf{e_3} + \alpha_2 \cdot \mathbf{e_4} \end{bmatrix},$$

$$\mathbf{u_t} = [v_t \quad e_t \quad 0 \quad 0 \quad w_t \quad \varepsilon_t], \mathbf{v_t} = [0 \quad \eta_t],$$

$\mathbf{e_j}, j = 1, \ldots, 6$, denotes a 1×6 row vector with element j equal to unity and all other elements equal to zero.

Random vectors $\mathbf{u_t}$, $\mathbf{v_t}$ are normally distributed and satisfy the following assumptions commonly assumed in the standard state space model:

$$E\left(\mathbf{v_t} \cdot \mathbf{v_s'}\right) = \begin{cases} \mathbf{\Sigma_{vv}} & \text{for } t = s, \\ \mathbf{0} & \text{for } t \neq s, \end{cases} \quad E\left(\mathbf{u_t} \cdot \mathbf{u_s'}\right) = \begin{cases} \mathbf{\Sigma_{uu}} & \text{for } t = s, \\ \mathbf{0} & \text{for } t \neq s, \end{cases} \tag{10}$$

$$E\left(\mathbf{v_t} \cdot \mathbf{u_s'}\right) = \mathbf{0} \quad \text{for all } t, s, \tag{11}$$

$$E\left(\mathbf{x_0} \cdot \mathbf{u_t'}\right) = E\left(\mathbf{x_0} \cdot \mathbf{v_t'}\right) = \mathbf{0} \quad \text{for all } t, \tag{12}$$

$$E(\mathbf{v_t}) = E(\mathbf{u_t}) = \mathbf{0} \quad \text{for all } t. \tag{13}$$

Quarterly seasonally adjusted data for unemployment rate U_t and GDP y_t in the Czech Republic from 1996 Q1 to 2015 Q4 were used for the observable variables in the vector $\mathbf{z_t}$. The relevant age structure for unemployed was chosen to be ‘from 24 to 74 years’. The GDP was also calendar adjusted and measured in chain linked volumes (2010) in millions euro. All such data is available at the Eurostat database.

2.2 Kalman Filter – Basic Version

The basic version of the Kalman filter algorithm summarized here for convenience is described e.g. in Harvey [11] or Hamilton [10]. The mean of the state vector $\mathbf{x_t}$ conditional on the information known in time $t-1$ is given by

$$\mathbf{x_{t|t-1}} = \mathbf{A} \cdot \mathbf{x_{t-1|t-1}}, \tag{14}$$

where $\mathbf{x_{t|t-1}} \equiv E(\mathbf{x_t}|\mathbf{\Omega_{t-1}})$, $\mathbf{x_{t-1|t-1}} \equiv E(\mathbf{x_{t-1}} \mid \mathbf{\Omega_{t-1}})$ and the information available in time $t-1$ is $\mathbf{\Omega_{t-1}} \equiv (\mathbf{z_1}, \ldots, \mathbf{z_{t-1}}, \mathbf{A}, \mathbf{D}, \mathbf{\Sigma_{uu}}, \mathbf{\Sigma_{vv}})$. The matrices $\mathbf{A}$, $\mathbf{D}$, $\mathbf{\Sigma_{uu}}$ and $\mathbf{\Sigma_{vv}}$ are assumed to be known in this chapter describing the Kalman filter algorithm despite the fact that they depend on unknown parameters $\boldsymbol{\theta} = (\phi_1, \phi_2, \alpha_0, \alpha_1, \alpha_2, \sigma_v, \sigma_w, \sigma_e, \sigma_\varepsilon, \sigma_\eta)$ in the model presented in Sect. 2.1. The estimation procedure of the parameter vector $\boldsymbol{\theta}$ will be described later in Sect. 2.4.

The prediction error covariance matrix is calculated as follows

$$\mathbf{P_{t|t-1}} \equiv E\Big[\big(\mathbf{x_t} - \mathbf{x_{t|t-1}}\big) \cdot \big(\mathbf{x_t} - \mathbf{x_{t|t-1}}\big)'|\mathbf{\Omega_{t-1}}\Big],$$

$$\mathbf{P_{t|t-1}} = \mathbf{A} \cdot E\Big[\big(\mathbf{x_{t-1}} - \mathbf{x_{t-1|t-1}}\big) \cdot \big(\mathbf{x_{t-1}} - \mathbf{x_{t-1|t-1}}\big)'|\mathbf{\Omega_{t-1}}\Big] \cdot \mathbf{A}' + E\big(\mathbf{u_t}\mathbf{u_t'}\big),$$

$$\mathbf{P_{t|t-1}} = \mathbf{A}\mathbf{P_{t-1}}\mathbf{A}' + \mathbf{\Sigma_{uu}}. \tag{15}$$

Distribution of the vector $(\, \mathbf{x_t'} \quad \mathbf{z_t'}\,)'$ conditional on $\mathbf{\Omega_{t-1}}$ is multivariate normal with mean $\left(\, \mathbf{x'_{t|t-1}} \quad (\mathbf{D} \cdot \mathbf{x_{t|t-1}})' \,\right)'$ and a covariance given by

$$E\left[\begin{pmatrix} \mathbf{x_t} - \mathbf{x_{t|t-1}} \\ \mathbf{D} \cdot \big(\mathbf{x_t} - \mathbf{x_{t|t-1}}\big) + \mathbf{v_t} \end{pmatrix}\begin{pmatrix} \mathbf{x_t} - \mathbf{x_{t|t-1}} \\ \mathbf{D} \cdot \big(\mathbf{x_t} - \mathbf{x_{t|t-1}}\big) + \mathbf{v_t} \end{pmatrix}'|\mathbf{\Omega_{t-1}}\right]$$
$$= \begin{pmatrix} \mathbf{P_{t|t-1}} & \mathbf{P_{t|t-1}}\mathbf{D}' \\ \mathbf{D}\mathbf{P_{t|t-1}} & \mathbf{D}\mathbf{P_{t|t-1}}\mathbf{D}' + \mathbf{\Sigma_{vv}} \end{pmatrix}.$$

Recall a generally known result that a conditional distribution of normally distributed vector $(\mathbf{x}'\mathbf{y}')'$ is also normal with mean and covariance given by

$$\boldsymbol{\mu}_{\mathbf{x|y}} = \boldsymbol{\mu}_{\mathbf{x}} + \mathbf{\Sigma_{xy}}\mathbf{\Sigma_{yy}^{-1}}\big(\mathbf{y} - \boldsymbol{\mu}_{\mathbf{y}}\big),$$
$$\mathbf{\Sigma_{xx|y}} = \mathbf{\Sigma_{xx}} - \mathbf{\Sigma_{xy}}\mathbf{\Sigma_{yy}^{-1}}\mathbf{\Sigma_{yx}},$$

where $\boldsymbol{\mu}_{\mathbf{x|y}} = E(\mathbf{x}|\mathbf{y})$, $\boldsymbol{\mu}_{\mathbf{x}} = E(\mathbf{x})$, $\boldsymbol{\mu}_{\mathbf{y}} = E(\mathbf{y})$,

$$\mathbf{\Sigma} = \begin{pmatrix} \mathbf{\Sigma_{xx}} & \mathbf{\Sigma_{xy}} \\ \mathbf{\Sigma_{yx}} & \mathbf{\Sigma_{yy}} \end{pmatrix} = E\left\{\begin{bmatrix} \mathbf{x} - \boldsymbol{\mu}_{\mathbf{x}} \\ \mathbf{y} - \boldsymbol{\mu}_{\mathbf{y}} \end{bmatrix} \cdot \big[\, (\mathbf{x} - \boldsymbol{\mu}_{\mathbf{x}})' \quad (\mathbf{y} - \boldsymbol{\mu}_{\mathbf{y}})' \,\big]\right\}.$$

Direct application of this result yields that $\mathbf{x_t}$ conditional on $\mathbf{z_t}$ is multivariate normal with mean

$$\mathbf{x_{t|t}} = \mathbf{x_{t|t-1}} + \mathbf{K_t}\big(\mathbf{z_t} - \mathbf{z_{t|t-1}}\big), \tag{16}$$

where

$$\mathbf{K_t} = \mathbf{P_{t|t-1}}\mathbf{D}'\big(\mathbf{D}\mathbf{P_{t|t-1}}\mathbf{D}' + \mathbf{\Sigma_{vv}}\big)^{-1}. \tag{17}$$

The covariance matrix is given by

$$\mathbf{P_t} = \mathbf{P_{t|t-1}} - \mathbf{P_{t|t-1}}\mathbf{D}'\left(\mathbf{D}\mathbf{P_{t|t-1}}\mathbf{D}' + \boldsymbol{\Sigma}_{\mathbf{vv}}\right)^{-1}\mathbf{D}\mathbf{P_{t|t-1}}. \tag{18}$$

Equations (14)–(18) together form the recursions of the Kalman filter algorithm.

2.3 Kalman Filter – Square Root Version

The basic version of the Kalman filter has disappointing numerical properties. For this reason, square root version of this algorithm is applied in this paper and briefly summarized here for convenience. More details can be found in Anderson and Moore [1] or Grewal and Andrews [9]. The square root version of the algorithm recursively calculates 'square roots' of the matrices $\mathbf{P_t}$ a $\mathbf{P_{t|t-1}}$, where the square root of a matrix is a lower triangular matrix $\mathbf{S_t}$ ($\mathbf{S_{t|t-1}}$) satisfying $\mathbf{P_t} = \mathbf{S_t} \cdot \mathbf{S}'_\mathbf{t}$ ($\mathbf{P_{t|t-1}} = \mathbf{S_{t|t-1}} \cdot \mathbf{S}'_{\mathbf{t|t-1}}$). The matrix $\mathbf{S_{t|t-1}}$ is calculated on the basis of the following equation

$$\begin{bmatrix} \mathbf{S}'_{\mathbf{t|t-1}} \\ \mathbf{0} \end{bmatrix} = \mathbf{T} \cdot \begin{bmatrix} \mathbf{S}'_{\mathbf{t-1}}\mathbf{A}' \\ \boldsymbol{\Sigma}^{\mathbf{1/2}'}_{\mathbf{uu}} \end{bmatrix}, \tag{19}$$

where $\mathbf{T}$ is a square orthogonal matrix ($\mathbf{T}' \cdot \mathbf{T} = \mathbf{I}$) ensuring that $\mathbf{S}'_{\mathbf{t|t-1}}$ is lower triangular.[1]

The matrix $\mathbf{S_{t|t-1}}$ is indeed a square root of $\mathbf{P_{t|t-1}}$ which can be easily verified as follows

$$\begin{aligned} \mathbf{S_{t|t-1}} \cdot \mathbf{S}'_{\mathbf{t|t-1}} &= \begin{bmatrix} \mathbf{S_{t|t-1}} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{S}'_{\mathbf{t|t-1}} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{A}\mathbf{S_{t-1}} & \boldsymbol{\Sigma}^{\mathbf{1/2}}_{\mathbf{uu}} \end{bmatrix} \cdot \mathbf{T}'\mathbf{T} \cdot \begin{bmatrix} \mathbf{S}'_{\mathbf{t-1}}\mathbf{A}' \\ \boldsymbol{\Sigma}^{\mathbf{1/2}'}_{\mathbf{uu}} \end{bmatrix} \\ &= \mathbf{A}\mathbf{S_{t-1}}\mathbf{S}'_{\mathbf{t-1}}\mathbf{A}' + \boldsymbol{\Sigma}^{\mathbf{1/2}}_{\mathbf{uu}}\boldsymbol{\Sigma}^{\mathbf{1/2}'}_{\mathbf{uu}} = \mathbf{A}\mathbf{P_{t-1}}\mathbf{A}' + \boldsymbol{\Sigma}_{\mathbf{uu}} = \mathbf{P_{t|t-1}}. \end{aligned}$$

The matrix $\mathbf{S_t}$ is calculated according to

$$\begin{bmatrix} \mathbf{F}^{1/2'}_{\mathbf{t|t-1}} & \tilde{\mathbf{K}}'_\mathbf{t} \\ \mathbf{0} & \mathbf{S}'_\mathbf{t} \end{bmatrix} = \bar{\mathbf{T}} \cdot \begin{bmatrix} \boldsymbol{\Sigma}^{1/2'}_{\mathbf{vv}} & \mathbf{0} \\ \mathbf{S}'_{\mathbf{t|t-1}}\mathbf{D}' & \mathbf{S}'_{\mathbf{t|t-1}} \end{bmatrix}, \tag{20}$$

where the matrix $\bar{\mathbf{T}}$ should have the same properties as the matrix $\mathbf{T}$ and the interpretation of the symbols $\mathbf{F}^{1/2'}_{\mathbf{t|t-1}}$, $\tilde{\mathbf{K}}'_\mathbf{t}$ will become clear from the following verification that the matrix $\mathbf{S_t}$ is a square root of the $\mathbf{P_t}$

[1] Specifically, the Matlab function *qr* was used for this purpose. $[\mathbf{QR}] = qr(\mathbf{A})$ calculates an upper triangular matrix $\mathbf{R}$ (with same dimensions as $\mathbf{A}$) and an orthogonal matrix $\mathbf{Q}$ such that $\mathbf{A} = \mathbf{Q} \cdot \mathbf{R}$, or $\mathbf{R} = \mathbf{Q}' \cdot \mathbf{A}$.

$$\begin{bmatrix} \mathbf{F}^{1/2}_{\mathbf{t|t-1}} & \mathbf{0} \\ \tilde{\mathbf{K}}_{\mathbf{t}} & \mathbf{S}_{\mathbf{t}} \end{bmatrix} \begin{bmatrix} \mathbf{F}^{1/2'}_{\mathbf{t|t-1}} & \tilde{\mathbf{K}}'_{\mathbf{t}} \\ \mathbf{0} & \mathbf{S}'_{\mathbf{t}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Sigma}^{1/2}_{\mathbf{vv}} & \mathbf{DS}_{\mathbf{t|t-1}} \\ \mathbf{0} & \mathbf{S}_{\mathbf{t|t-1}} \end{bmatrix} \cdot \bar{\mathbf{T}}'\bar{\mathbf{T}} \cdot \begin{bmatrix} \boldsymbol{\Sigma}^{1/2'}_{\mathbf{vv}} & \mathbf{0} \\ \mathbf{S}'_{\mathbf{t|t-1}}\mathbf{D}' & \mathbf{S}'_{\mathbf{t|t-1}} \end{bmatrix},$$

or

$$\begin{bmatrix} \mathbf{F}_{\mathbf{t|t-1}} & \mathbf{F}^{1/2}_{\mathbf{t|t-1}}\tilde{\mathbf{K}}'_{\mathbf{t}} \\ \tilde{\mathbf{K}}_{\mathbf{t}}\mathbf{F}^{1/2'}_{\mathbf{t|t-1}} & \tilde{\mathbf{K}}_{\mathbf{t}}\tilde{\mathbf{K}}'_{\mathbf{t}} + \mathbf{S}_{\mathbf{t}}\mathbf{S}'_{\mathbf{t}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Sigma}_{\mathbf{vv}} + \mathbf{DS}_{\mathbf{t|t-1}}\mathbf{S}'_{\mathbf{t|t-1}}\mathbf{D}' & \mathbf{D}'\mathbf{S}_{\mathbf{t|t-1}}\mathbf{S}'_{\mathbf{t|t-1}} \\ \mathbf{S}_{\mathbf{t|t-1}}\mathbf{S}'_{\mathbf{t|t-1}}\mathbf{D} & \mathbf{S}_{\mathbf{t|t-1}}\mathbf{S}'_{\mathbf{t|t-1}} \end{bmatrix}.$$

Comparison of the corresponding submatrices in the left yields

$$\mathbf{F}_{\mathbf{t|t-1}} = \left(\mathbf{DP}_{\mathbf{t|t-1}}\mathbf{D}' + \boldsymbol{\Sigma}_{\mathbf{vv}}\right),$$

$$\tilde{\mathbf{K}}_{\mathbf{t}} = \mathbf{P}_{\mathbf{t|t-1}}\mathbf{D}\left(\mathbf{F}^{1/2'}_{\mathbf{t|t-1}}\right)^{-1}.$$

Comparing the matrices in the lower right block reveals that

$$\begin{aligned} \mathbf{S}_{\mathbf{t}}\mathbf{S}'_{\mathbf{t}} &= \mathbf{S}_{\mathbf{t|t-1}}\mathbf{S}'_{\mathbf{t|t-1}} - \tilde{\mathbf{K}}_{\mathbf{t}}\tilde{\mathbf{K}}'_{\mathbf{t}} \\ &= \mathbf{S}_{\mathbf{t|t-1}}\mathbf{S}'_{\mathbf{t|t-1}} - \mathbf{P}_{\mathbf{t|t-1}}\mathbf{D}\left(\mathbf{F}^{1/2'}_{\mathbf{t|t-1}}\right)^{-1}\left(\mathbf{F}^{1/2}_{\mathbf{t|t-1}}\right)^{-1}\mathbf{D}'\mathbf{P}_{\mathbf{t|t-1}} \\ &= \mathbf{P}_{\mathbf{t|t-1}} - \mathbf{P}_{\mathbf{t|t-1}}\mathbf{DF}^{-1}_{\mathbf{t|t-1}}\mathbf{D}'\mathbf{P}_{\mathbf{t|t-1}} \\ &= \mathbf{P}_{\mathbf{t}}. \end{aligned}$$

Vectors $\mathbf{x}_{\mathbf{t|t-1}}$ and $\mathbf{x}_{\mathbf{t|t}}$ are calculated in the same way as in the basic form of the Kalman filter (see Eqs. 14 and 16), but the matrix $\mathbf{K}_{\mathbf{t}}$ is calculated as $\mathbf{K}_{\mathbf{t}} = \tilde{\mathbf{K}}_{\mathbf{t}} \cdot \left(\mathbf{F}^{1/2}_{\mathbf{t|t-1}}\right)^{-1}$. This indeed corresponds to the way it is calculated in the basic version of the Kalman fiter which can be seen easily as follows

$$\begin{aligned} \mathbf{K}_{\mathbf{t}} &= \tilde{\mathbf{K}}_{\mathbf{t}} \cdot \left(\mathbf{F}^{1/2}_{\mathbf{t|t-1}}\right)^{-1} \\ &= \mathbf{P}_{\mathbf{t|t-1}}\mathbf{D}\left(\mathbf{F}^{1/2'}_{\mathbf{t|t-1}}\right)^{-1}\left(\mathbf{F}^{1/2}_{\mathbf{t|t-1}}\right)^{-1} \\ &= \mathbf{P}_{\mathbf{t|t-1}}\mathbf{D}\left(\mathbf{F}_{\mathbf{t|t-1}}\right)^{-1}. \end{aligned}$$

2.4 Likelihood Function

The likelihood function was computed by the method described in Harvey [11] or Hamilton [10] which is briefly summarized here for convenience. Let us denote $\mathbf{Z}_{\mathbf{T}} = \left(\mathbf{z}'_{\mathbf{1}}, \ldots, \mathbf{z}'_{\mathbf{T}}\right)'$ the column vector with n rows, where n is a multiple of number of observations T and a number of observed variables. The multivariate density of observed data $\mathbf{z}_{\mathbf{t}}$, $t = 1, \ldots, T$ will be denoted by $f_0(\mathbf{Z}_{\mathbf{T}})$ and is a member of

$\{f(\mathbf{Z_T}|\boldsymbol{\theta}), \boldsymbol{\theta} \in \boldsymbol{\Theta}\}$. Hence, $f_0(\mathbf{Z_T}) = f(\mathbf{Z_T}|\boldsymbol{\theta_0})$, where $\boldsymbol{\theta_0}$ are true (unknown) parameter values. For the model described in Sect. 2.1, the vector $\boldsymbol{\theta}$ is given by $\boldsymbol{\theta} = \left(\phi_1, \phi_2, \alpha_0, \alpha_1, \alpha_2, \sigma_v, \sigma_w, \sigma_e, \sigma_\varepsilon, \sigma_\eta\right)$.

The density $f(\mathbf{Z_T} \mid \boldsymbol{\theta})$ can be viewed as a function of $\boldsymbol{\theta}$ for given data $\mathbf{Z_T}$, i.e. $L(\boldsymbol{\theta} \mid \mathbf{Z_T}) \equiv f(\mathbf{Z_T} \mid \boldsymbol{\theta})$, where the function $L(\boldsymbol{\theta} \mid \mathbf{Z_T})$ is called a likelihood function. The method of maximum likelihood estimates the unknown parameter vector $\boldsymbol{\theta_0}$ by maximizing the likelihood function

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmax}}[L(\theta|\mathbf{Z_T})] \tag{21}$$

where $\hat{\boldsymbol{\theta}}$ is the maximum likelihood estimate of the parameter vector $\boldsymbol{\theta_0}$.

It can be easily shown that the likelihood function can be written as follows

$$L(\boldsymbol{\theta}|\mathbf{Z_T}) = \prod_{t=1}^{T} f(\mathbf{z_t}|\mathbf{Z_{t-1}}, \boldsymbol{\theta}).$$

The vector $\mathbf{z_t}$ is normally distributed in our case. Hence, the distribution of $\mathbf{z_t}$ conditional on $\mathbf{Z_{t-1}}$ is also normal. The mean and covariance of this conditional distribution are obtained from the Kalman filter as follows

$$\mathbf{z_{t|t-1}} \equiv E(\mathbf{z_t}|\boldsymbol{\Omega}_{\mathbf{t-1}}) = \mathbf{D} \cdot \mathbf{x_{t|t-1}}, \tag{22}$$

$$\mathbf{F_{t|t-1}} \equiv E\left[\left(\mathbf{z_t} - \mathbf{z_{t|t-1}}\right)\left(\mathbf{z_t} - \mathbf{z_{t|t-1}}\right)'|\boldsymbol{\Omega}_{\mathbf{t-1}}\right] = \mathbf{D}\mathbf{P_{t|t-1}}\mathbf{D}' + \boldsymbol{\Sigma}_{\mathbf{vv}}. \tag{23}$$

The likelihood function in this case is given by

$$L(\boldsymbol{\theta}|\mathbf{Z_T}) = \prod_{t=1}^{T}\left[\frac{1}{(2\pi)^{\frac{k}{2}}\left|\mathbf{F_{t|t-1}}\right|^{\frac{1}{2}}}\exp\left(-\frac{1}{2}\tilde{\mathbf{z}}_{\mathbf{t}}'\mathbf{F}_{\mathbf{t|t-1}}^{-1}\tilde{\mathbf{z}}_{\mathbf{t}}\right)\right],$$

where k is the number of observed variables and $\tilde{\mathbf{z}}_{\mathbf{t}} = \mathbf{z_t} - \mathbf{z_{t|t-1}}$ is so called innovation.

From a numerical point of view, it is easier to maximize the log-likelihood function which is calculated as follows

$$l(\boldsymbol{\theta}|\mathbf{Z_T}) \equiv \ln L(\boldsymbol{\theta}|\mathbf{Z_T}) = -\frac{T \cdot k}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left[\ln\left|\mathbf{F_{t|t-1}}\right| + \left(\tilde{\mathbf{z}}_{\mathbf{t}}'\mathbf{F}_{\mathbf{t|t-1}}^{-1}\tilde{\mathbf{z}}_{\mathbf{t}}\right)\right]. \tag{24}$$

This function was maximized numerically using standard numerical optimization procedures implemented in Matlab in order to find maximum likelihood estimate $\hat{\boldsymbol{\theta}}$.

3 Empirical Application

3.1 Results of Econometric Estimation

Econometric estimation was performed in Matlab by maximization of the likelihood function (24). The obtained results are summarized in the following (Tables 1 and 2).

Table 1. Econometric estimates—GDP decomposition

	ϕ_1	ϕ_2	σ_v	σ_e	σ_w
Estimate	1.5396	−0.6345	0.0026	0.0055	0.0022
Standard error	0.0672	0.0661	0.0016	0.0010	0.0006

Table 2. Econometric estimates—unemployment decomposition

	α_0	α_1	α_2	σ_ε	σ_η
Estimate	−0.1170	−0.1649	−0.1117	0.0015	0.0002
Standard error	0.0425	0.0568	0.0364	0.0005	0.0011

The estimated value $\sigma_e = 0.0055$ indicates that a significant portion of quarter-to-quarter innovations in real GDP is cyclical. Similar results were obtained for the U.S. economy (Clark [6], Kim, Nelson [12]). Nonetheless, the estimated standard error $\sigma_w = 0.0022$ representing the variability of the GDP (long-run) growth is approximately 10 times higher than that reported by Clark or Kim and Nelson.

One possible explanation for why the U.S. economic long-run growth is less volatile than the long-run growth in the Czech Republic is the process of economic transformation. The Czech economy had been opening to the rest of the world. Many international trade barriers had been removed by the entrance to the European Union. Lots of economic reforms had been realized. For these reasons, volatility of the long-run growth of the economy in transition is higher than the volatility of a stable economy.

Negative values of the parameters α_i, $i = 0, 1, 2$ confirm an inverse relationship between output and unemployment referred to as Okun's law.

3.2 Economic Discussion

The following Fig. 1 illustrates the decomposition of the log of real GDP to its trend and a cyclical component (output gap). There was a sharp decline in the output gap in 2008 and 2009. Nonetheless, it has recovered after this initial shock and has been even positive since 2014 Q1. This finding is interesting as it is in contrast with earlier business-cycle studies for the U.S. economy. Watson [16], Kim and Nelson [12] and Clark [6] attribute most of the variation of U.S. output to the cyclical component.

The graph in the left suggests that there is a change in the GDP trend due to the current crisis. Similar results are found by Perron and Wada [15] who emphasized the importance of changes in the slope of the trend. This suggests that the huge output loss

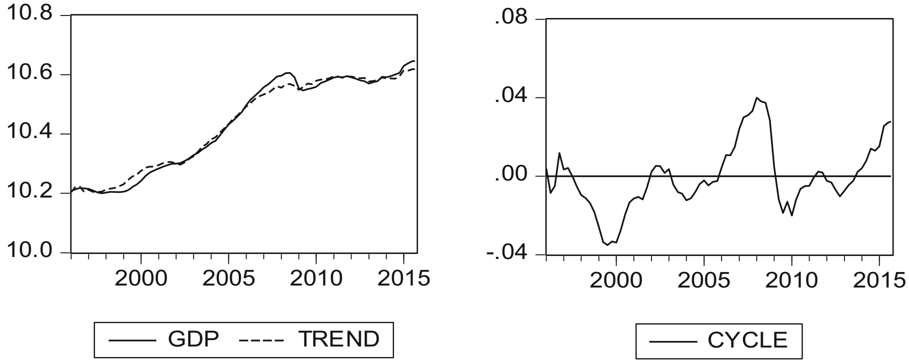

Fig. 1. Log of real GDP y_t and its trend n_t and a cycle x_t component

induced by the crisis is permanent and not only transitory. While the trend was upward-sloping from 1996 to 2008, it is practically constant from 2009 to 2015. This result can also be seen in the Fig. 2 which shows that the quarter-to-quarter growth of the GDP trend component g_t has been low since 2009. The mean of the variable g_t in the pre-crisis time period from 1996 to 2008 is 0.0081 which is quite high when compared to the corresponding value 0.0018 for the post-crisis period from 2009 to 2015. These results are in line with other empirical studies analyzing the impact of the current global economic crisis (Barro [3], Ball [2]).

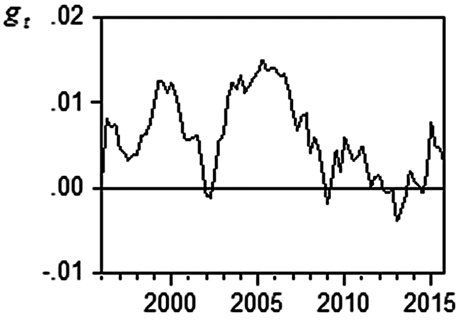

Fig. 2. Long-run growth of the GDP trend component g_t

These findings suggest an adverse permanent influence of the crisis on the long-run economic growth. Nonetheless, this is only a suggestion. The rigorous evaluation of the influence of the crisis on the long-run growth is left for future research. Such a research would apply the growth theory according to which the growth rate depends on its determinants. Changing these determinants leads to a change in the long-run growth rate. The current economic crisis can be considered to be just one of many determinants of the long-run growth rate. Nonetheless, it is probably the case that the current economic crisis is indeed the most important factor which caused the decreased values of the long-run growth g_t after 2008.

The Fig. 3 shows the trend-cycle decomposition of the unemployment rate.

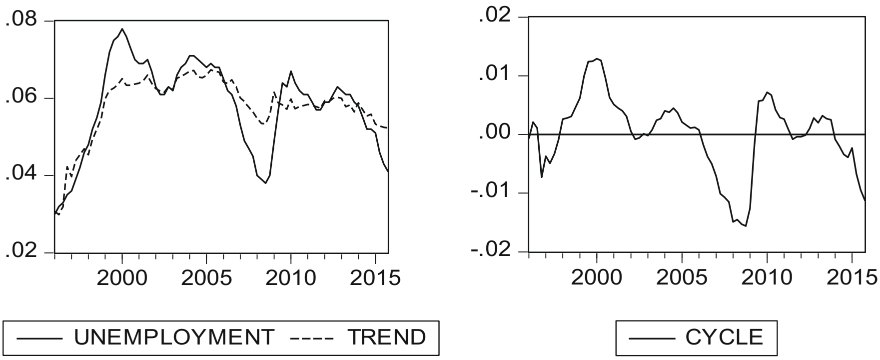

Fig. 3. Unemployment rate U_t and its trend L_t and a cycle C_t component

The cyclical component increased substantially in the beginning of the crisis in 2009. Since then, however, the cyclical component of the unemployment rate has been decreasing steadily. The trend component has been practically constant since the beginning of the crisis which suggests that possible hysteresis effects haven't played important role yet.

4 Conclusion

The paper applies state space methodology in order to perform trend-cycle decomposition of the GDP and unemployment rate in the Czech Republic. The important finding is that the long-run growth in the Czech Republic has been highly volatile and that there has been a dramatic decrease of this long-run growth since 2008. Similar results of a huge long-term damage to output were found by Ball [2] as well as by many others cited in Ball's influential paper. The trend of unemployment rate has not changed much since the beginning of the crisis while the cyclical component increased sharply at the beginning of the crisis and has been decreasing steadily since then.

The model could be expanded for example by relaxing the common presumption of no correlation between the shocks to the trend and the cycle (Morley et al. [13]). Possible parameter instability due to the economic crisis could also be taken into account by applying regime-switching methodology as by Cerra and Saxena [4]. Detailed analysis along the lines of the growth theory as in Barro [3], or Durlauf, Helliwell, Raj [8] could be applied in order to confirm the suggested hypothesis that the observed decline in the growth of the long-run trend is caused by the current economic crisis.

Acknowledgements. Financial support of VŠE IGA IG403036 is gratefully acknowledged by the author. Paper was processed with contribution of long term support of scientific work on Faculty of Informatics and Statistics, University of Economics, Prague (IP 400040).

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http://www.springer.com/978-3-319-67620-3

Applied Computational Intelligence and Mathematical Methods
Computational Methods in Systems and Software 2017, vol. 2
Silhavy, R.; Silhavy, P.; Prokopova, Z. (Eds.)
2018, XIV, 394 p. 142 illus., Softcover
ISBN: 978-3-319-67620-3